Exergy Optimization Applied to Linear Parabolic Solar Collectors

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Abstract

A new method of optimization on linear parabolic solar collectors using exergy analysis is presented. A comprehensive mathematical modeling of thermal and optical performance is simulated and geometrical and thermodynamic parameters were assumed as optimization variables. By applying a derived expression for exergy efficiency, exergy losses were generated and the optimum design and operating conditions, were investigated. The objective function (exergy efficiency) along with constraint equations constitutes a four-degree freedom optimization problem. Using Lagrange multipliers method, the optimization procedure was applied to a typical collector and the optimum design point was extracted. The optimum values of collector inlet temperature, oil mass flow rate, concentration ratio and glass envelope diameter are calculated simultaneously by numerical solution of a highly non-linear equations system. To study the effect of changes in optimization variables on the collected exergy, the sensitivity of optimization to changes in collector parameters and operating conditions is evaluated and variation of exergy fractions at this point are studied.

Keywords: Exergy - Optimization - Linear Parabolic Collectors - Solar Energy

Introduction

Linear parabolic collectors, known as parabolic trough collectors, are generally used in energy conversion and power generation. This type of collector, as depicted in Fig. 1, is capable of supplying thermal energy over a wide range of temperatures up to about 305° C. In those cases where a solar collector is used to drive an energy conversion device, such as an engine, it is the exergy collection capability rather than energy collection capability that is the true measure of the potential of the collector to perform the desired function. To gain the maximum exergy (or to minimize irreversibilities), design and performance parameters of collector are to be optimized. The basic elements of such a collector are: *i*) parabolic concentrator, *ii*) receiver at the focal axis of parabola. The receiver has a selective coating and a glass envelope around it. The primary function of the receiver is to absorb and transfer the concentrated energy to the fluid flowing through it.

The temperature difference between the absorbing surface and the surroundings will return some of the collected energy back to the surroundings, i.e. lost. The variables affecting collector efficiency fall into two groups: operating conditions and geometrical structure. The optimum operating conditions (flow rate, operating temperature) are defined as that by which the maximum value of exergy efficiency would be obtained. The geometrical design parameters are gap width, concentration ratio and rim angle whose optimum values enable one to design a solar collector that is approximately optimal for a range of operating conditions.

Effective utilization of solar energy will be achieved by constructing a collector system which operates with least wasteful processes. It is found that existing analysis techniques for optimization and modeling collector's performance incorporated restrictive assumptions.

In most previous studies, the variation of exergy efficiency against collector inlet temperature and mass flow rate are investigated. These studies show that in a specific inlet temperature and mass flow rate, collector gains maximum exergy [1-12].

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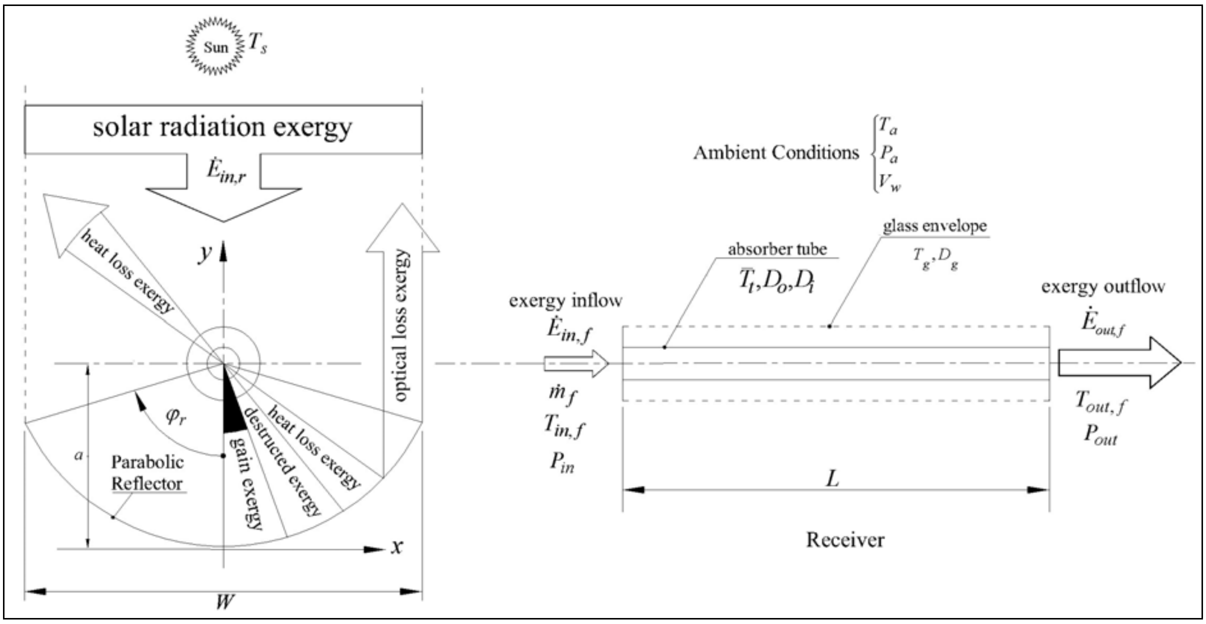

Fig. 1: Exergy flows and dimensions in a typical linear parabolic solar collector.

In some previous studies specially for this type of collector, the main design objective has been the maximization of thermal efficiency by selecting optimum concentration ratio [13-16].

The results of these investigations were not sufficiently functional and comprehensive to serve as design guide to collect maximum available exergy. They consider only a narrow range of collector parameters and operating conditions, and overall heat loss coefficient assumed as constant. Since the absorber surface temperature varies in a real case, in this paper the overall heat loss coefficient U_l between outer surface tube temperature and ambient temperature is supposed as a variable parameter and was modeled in a calculation procedure.

In the present study, a more general and comprehensive approach is taken and a comprehensive and general design and optimization method for design and analysis of linear parabolic solar collectors is presented. The presented design method comprises of an objective design approach, optical modeling, thermal modeling and optimization procedure. It can be used in any design conditions for any given set of constraints and design requirements.

The efficiency of the collector should be maximized if such a step does not increase the collector cost. This optimization does not increase cost because it does not focus on insulation, tracking mode, properties of material including reflectance and absorptance, absorber shape and evacuated or non-evacuated glass envelope. This method permits cost savings through optimization, because lower collector surface area is required for a specific function.

Exergy Analysis

Application of exergy analysis to solar parabolic concentrators helps designers to achieve an optimum design and gives direction to decrease exergy losses. Exergy concept is one of the two ways to the second law analysis, and entropy generation from irreversibilities is the other method. However, both techniques fundamentally give identical results. Decreasing exergy destructed during thermodynamic processes can be achieved by the view point of maximizing exergy efficiency.

By applying exergy balance on a solar collector (shown in Fig. 1), exergy efficiency can be derived and the shares of irreversible factors are defined as well. Exergy balance on a parabolic solar collector, as depicted in Fig. 1, can be generally expressed as [7]:

$$\Sigma\dot{E}_{in}-\Sigma\dot{E}_{out}-\Sigma\dot{E}_{loss}-\Sigma\dot{E}_{change}-\Sigma\dot{E}_{des}=0 \quad (1)$$

The exergy input rate ($\sum\dot{E}_{in}$) includes the exergy accompanying mass flow rate and the exergy of solar radiation and the exergy out rate ($\sum\dot{E}_{out}$) just includes the exergy mass flow. The gain exergy rate ($\dot{E}_{gain}$) is the exergy accumulated by fluid flow through the absorber tube and is expressed as:

$$\dot{E}_{gain}=\dot{E}_{out,f}-\dot{E}_{in,f} \quad (2)$$

and exergy efficiency is the ratio of gain exergy to solar radiation exergy, then:

$$\eta_E=\frac{\dot{E}_{gain}}{\dot{E}_{in,r}} \quad (3)$$

Defining a non-dimensional parameter as $\overset{*}{\dot{E}}=\dot{E}/\dot{E}_{in,r}$ which represents the ratio of exergy to the incoming radiation exergy from the sun, and substituting eq (1) and (2) into eq (3), exergy efficiency equation becomes:

$$\eta_E=\overset{*}{\dot{E}}_{gain}=1-\sum\overset{*}{\dot{E}}_{loss}-\sum\overset{*}{\dot{E}}_{change}-\sum\overset{*}{\dot{E}}_{des} \quad (4)$$

In general, exergy can be exchanged in two ways in solar collectors: the first, by fluid flow and the second by heat transfer. The exergy accompanying an incompressible fluid flow is as:

$$\dot{E}_f=\dot{m}_f c_p(T-T_a-T_a\ln\frac{T}{T_a})+\frac{\dot{m}_f\Delta P}{\rho} \quad (5)$$

where T is fluid temperature and ΔP is pressure difference between the fluid and ambient pressure. The transferred exergy by heat transfer ($\dot{q}$) between hot and cold temperatures (T_h and T_c) is defined as:

$$\dot{E}_{\dot{q}}=\int_{T_c}^{T_h}\dot{q}\frac{T_a}{T^2}dT \quad (6)$$

According to Petela's theory, the exact exergy income by solar radiation for a typical collector with surface area of A_c becomes:

$$\dot{E}_{in,r}=I_b A_c\eta_p \quad (7)$$

η_p is Petela's efficiency of converting radiation energy (i.e. $I_b A_c$) into work as[17]:

$$\eta_p=1-\frac{4}{3}\frac{T_a}{T_s}+\frac{1}{3}\left(\frac{T_a}{T_s}\right)^4 \quad (8)$$

where T_s is the sun temperature. Considering the exergy at inlet and outlet of the absorber tube and Eq. (3), the exergy efficiency is expressed as:

$$\eta_E=\frac{\dot{m}_f}{I_b A_c\eta_P}\left[c_p\left(T_{out}-T_{in}-T_a\,ln(\frac{T_{out}}{T_{in}})\right)-\frac{\Delta P}{\rho}\right] \quad (9)$$

the exergy efficiency should be expressed in terms of lost and destructed exergy to illustrate which exergy fractions are major, and consequently take direction to decrease them. Exergy loss rate is the amount of exergy that a thermodynamic system loses in processes. In fact, it is the exergy leakage rate out to the surroundings due to optical errors and heat transfer to ambient in a solar collector and is undesirable:

$$\sum\overset{*}{\dot{E}}_{loss}=\overset{*}{\dot{E}}_{l,opt}+\overset{*}{\dot{E}}_{l,th} \quad (10)$$

Due to optical error and surface properties of the absorber tube in such a collector, parts of solar radiation do not reach to the absorber tube. Then the fraction of optical exergy rate is:

$$\overset{*}{\dot{E}}_{l,opt}=\frac{\dot{E}_{l,opt}}{\dot{E}_{in,r}}=\frac{(1-\eta_o)\dot{E}_{in,r}}{\dot{E}_{in,r}}=1-\eta_o \quad (11)$$

where η_o is optical efficiency as $\eta_o=S/I_b$. The heat leakage from absorber tube to ambient is defined as $\dot{Q}_{l,th}=U_l A_t(\overline{T}_t-T_a)$ and its corresponding exergy loss fraction becomes:

$$\overset{*}{\dot{E}}_{l,th}=\frac{\int_{T_a}^{\overline{T}_t}\dot{Q}_{l,th}\frac{T_a}{T^2}dT}{I_b A_c\eta_p}=\frac{U_l A_t(\overline{T}_t-T_a)^2}{I_b A_c\eta_p\overline{T}_t} \quad (12)$$

the exergy change rate in parabolic solar collectors is excluded to exergy change in the substance of fluid flow and absorber tube whose temperature values change. Studying the collector in steady state conditions, $\sum\overset{*}{\dot{E}}_{change}=0$; however, exergy change rate of the tube and fluid can be approximately neglected in every condition due to low values.

$$\sum \overset{*}{\dot{E}}_{change} = \frac{d}{dt}(\overset{*}{\dot{E}}_t + \overset{*}{\dot{E}}_f) = 0 \quad (13)$$

Exergy destruction is caused by irreversibilities in the system and there are two ways of exergy destruction in solar collectors; exergy destructed due to friction of viscous fluid and exergy destructed due to heat transfer processes. Since the viscous fluid causes pressure drop between inlet and outlet of the tube, considering correspondent entropy generation, exergy destructed during this process can be stated as:

$$\dot{E}_{des,\Delta P} = T_a \frac{\dot{m}_f \Delta P}{\rho} \frac{\ln(T_{out}/T_{in})}{T_{out} - T_{in}} \quad (14)$$

Exergy also is destructed while heat is transferred from hot to cold temperatures. There are two heat transfer processes in the collector; 1) heat transfer of solar energy absorbed by the surface of the tube, 2) heat transfer conduction from outer tube surface to fluid flow. Applying Eq (6) for the heat transfer processes from sun temperature (T_s) to absorber surface temperature $(\overline{T}_t)$ and then to fluid flow at temperature of (T_f), exergy destructed is stated as:

$$\dot{E}_{des,th} = \underbrace{\int_{\overline{T}_t}^{T_s} \dot{Q}_u \frac{T_a}{T^2} dT}_{absorption} + \underbrace{\int_{T_f}^{\overline{T}_t} \dot{Q}_u \frac{T_a}{T^2} dT}_{conduction} \quad (15)$$

where $\dot{Q}_u$ is useful energy rate added to the fluid flow energy. Considering useful energy rate as $\dot{Q}_u = \eta_o I_b A_c$, the exergy destructed rate due to heat absorption and conduction processes are defined respectively as:

$$\dot{E}_{des,abs} = \eta_o I_b A_c T_a (1/\overline{T}_t - 1/T_s) \quad (16)$$

$$\dot{E}_{des,cond} = \dot{m}_f c_p T_a \left(ln\left(\frac{T_{out}}{T_{in}}\right) - \frac{(T_{out} - T_{in})}{\overline{T}_t} \right) \quad (17)$$

then, the exergy destruction is combined of three terms:

$$\sum \overset{*}{\dot{E}}_{des} = \frac{\dot{E}_{des,\Delta P} + \dot{E}_{des,abs} + \dot{E}_{des,cond}}{\dot{E}_{in,r}} \quad (18)$$

Substituting Eqs. (11-14) ,(16-18) into Eq. (4), the expression of exergy efficiency can be presented as:

$$\eta_E = 1 - \left[\underbrace{(1-\eta_o)}_{\dot{E}^*_{l,opt}} + \underbrace{\frac{U_l A_t}{I_b A_c \eta_P} \frac{(\overline{T}_t - T_a)^2}{\overline{T}_t}}_{\dot{E}^*_{l,th}} + \underbrace{\frac{T_a \dot{m}_f \Delta P}{I_b A_c \eta_P \rho} \frac{ln(T_{out}/T_{in})}{T_{out} - T_{in}}}_{\dot{E}^*_{des,\Delta P}} + \underbrace{\frac{\eta_o T_a}{\eta_P} \left(\frac{1}{\overline{T}} - \frac{1}{T_s}\right)}_{\dot{E}^*_{des,abs}} + \underbrace{\frac{\dot{m}_f c_p T_a}{I_b A_c \eta_P} \left(ln\frac{T_{out}}{T_{in}} - \frac{T_{out} - T_{in}}{\overline{T}_t} \right)}_{\dot{E}^*_{des,cond}} \right] \quad (19)$$

Using Eq. (19) as exergy efficiency illustrates which parts of losses or destruction have greater influence on the value of efficiency.

Optical and Geometrical Modeling

The optical analysis of solar collectors with parabolic reflectors must take into account many different effects, such optical properties of materials, relative size of absorber and concentrator and the type of tracking and corresponding losses. In this study, an optical design method is used which can be compatible for design analysis of linear parabolic collector for different design conditions.

We consider the optical performance analysis of a parabolic concentrating collector. The geometrical parameters of a collector whose concentrator has an aperture W, length L, rim angle ϕ_r are mathematically correlated.

The concentration ratio is the ratio of the effective area of the aperture to the surface area of the absorber and is given as:

$$C = \frac{W - D_o}{\pi D_o} \quad (20)$$

As C increases, the heat loss per aperture decreases, but the fraction of reflected solar radiation absorbed by receiver also decreases. Conceptually, it is more convenient to optimize the concentration ratio by fixing the receiver size and changing the aperture area. So, the outer diameter of absorber tube (D_o), is a fixed parameter in the mathematical modeling.

Since concentrator aperture width (W) has direct effect on the concentration ratio, the collector surface area is assumed as a given value and relative dimensions of concentrator width and length can be variable:

$$A_c = L.W \tag{21}$$

A linear parabolic collector is oriented with its focal axis and because of the optical system, certain losses are introduced. The combined effect of all such losses is indicated in through the introduction of a term called the optical efficiency. The optical efficiency is the fraction of solar radiation incident on the aperture of the collector which is absorbed at the surface of the absorber tube.

$$\eta_o = S / I_b \tag{22}$$

S is the absorbed flux and I_b is the beam radiation normally incident on the aperture. The value of absorbed flux is as following:

$$S = I_b \rho \gamma (\tau\alpha)_b + I_b (\tau\alpha)_b (D_o /(W - D_o)) \tag{23}$$

Optical efficiency embodies many important concentrator optical properties including mirror surface reflectance (ρ), receiver cover transmittance (τ), and absorber surface absorptance (α). It also includes the effects of solar beam intercept factor (γ), which includes all optical errors. The decrease in optical efficiency due to errors can be determined by analyzing the effect of errors on the intercept factor (γ). Several investigators have studied the errors affecting the optical performance of trough parabolic solar collectors [13,15]. The value of γ is defined based on the universal error parameters i.e.

$\gamma = \gamma(\sigma^*, \beta^*, d^*, \varphi_r)$ where:

$$\begin{cases} \sigma^* = \sigma_{tot} C \\ \beta^* = \beta.C \\ d^* = d_r / D_o \end{cases} \tag{24}$$

σ_{tot} is compound of standard deviations of random errors including energy distribution standard deviation of the sun's rays at normal incidence, the distribution of local slope errors at normal incidence, diffusivity of the reflective material at normal incidence. β is standard deviation of reflector misalignment and tracking error angle, and d_r is concentrator miss-location error parameter. The function of γ is expressed as following:

$$\gamma = \frac{1+\cos\varphi_r}{2\sin\varphi_r} \int_0^{\varphi_r} \frac{erf(\psi_1) - erf(-\psi_2)}{1+Cos\varphi} d\varphi \tag{25}$$

$$\begin{cases} \psi_1 = \dfrac{Sin\varphi_r (1+Cos\varphi)(1-2d^* Sin\varphi) - \pi\beta^* (1+Cos\varphi_r)}{\sqrt{2}\pi\sigma^* (1+Cos\varphi_r)}, \\ \psi_2 = \dfrac{Sin\varphi_r (1+Cos\varphi)(1+2d^* Sin\varphi) + \pi\beta^* (1+Cos\varphi_r)}{\sqrt{2}\pi\sigma^* (1+Cos\varphi_r)} \end{cases} \tag{25-a}$$

For different design and manufacturer conditions, the effect of gross error that may result from poor manufacture, assembly or lesser technological capabilities is different. So, there are differences in design and manufacture between industrialized and developing countries. The values of σ_{tot}, β, d_r were given in references for deferent design environment i.e., industrialized or developing countries.

Lower concentration ratio quantitatively permits a greater amount of the reflected energy to be intercepted by the absorber tube. However, low concentration ratios will also increase the absorber tube surface area relative to the aperture area, and result in increased heat loss per unit area of aperture. Conversely, high concentration ratios reduce heat losses but increase optical losses, i.e., the fraction of the incident solar radiation that is intercepted by the absorber will decrease. The optical characteristics of collector are directly coupled to the thermal analysis, because the variation of concentration ratio (C) affects both optical and thermal performance of the collector. Hence, the optical efficiency cannot be modeled and analyzed independently without knowledge of the thermal design and vice versa.

Thermal modeling

The energy balance on the absorber tube yields the useful energy rate for a steady state condition:

$$\dot{Q}_u = F_R (W - D_o) L \left[S - U_l (T_{in,f} - T_a)/C \right] \tag{26}$$

Where U_l is overall heat loss coefficient of the collector receiver surface area and it depends on the magnitude of convective, radiation and conduction losses, which in turn depends on the collector operating temperature $(\overline{T}_t)$ relative to the environment (T_a) [18].

The useful heat gain can also be defined on the base of fluid difference temperatures as:

$$\dot{Q}_u = \dot{m}_f c_p (T_{f,out} - T_{f,in}) \tag{27}$$

F_R is the heat removal factor and F' is the collector efficiency factor defined as:

$$F_R = \frac{\dot{m}_f c_p}{\pi D_o L U_l}\left(1 - \exp\left(-\frac{F' \pi D_o U_l L}{\dot{m}_f c_p}\right)\right) \tag{28}$$

F_R is a measure of the efficiency of the receiver when viewed as a heat exchanger, that is, the effectiveness with which the absorbed radiation energy is transferred to the working fluid. Its value is governed by the working fluid flow rate and its properties as well as the thermal properties of the receiver material.

$$F' = 1/(U_l(1/U_l + 1/U_c)) \tag{29}$$

Considering the absorber tube and the glass cover around it, overall heat loss coefficient U_l based on convection and radiation losses can be modeled.

U_c is overall heat gain coefficient between mean absorber temperature $(\overline{T}_t)$ and local fluid flow temperature (T_f). U_c is calculated by conduction and convection heat transfer modeling from outer surface of absorber tube towards the tube center.

There are two heat transfer coefficients: heat transfer coefficient between absorber tube and glass cover (properties are evaluated at the mean temperature $(\overline{T}_t + T_c)/2$), and heat transfer coefficient on outside surface of glass cover (properties are evaluated at the mean temperature $(T_a + T_c)/2$)[17-19].

Calculation of U_l and U_c is complicated and contains several equations and parameters, then, the expressions of them are summarized as functions of influencing variables and parameters as:

$$U_l = U_l\left(\frac{D_o}{D_c}, \varepsilon_p, \varepsilon_c, T_a, \overline{T}_t, V_w, \text{air properties}\right) \tag{30}$$

$$U_c = U_c(D_o / D_i, \dot{m}_f, L, \text{oil properties}) \tag{31}$$

While the absorber tube diameter is fixed, the receiver-glazing diameter is sized to minimize conductive/convection/radiation losses. The elimination or reduction of conduction and natural convection losses can significantly improve the performance of a collector. Too small a glass diameter (small gap) results in excessive conduction losses, whereas too large a glass diameter (large gap) results in excessive convection losses [20].

Wind velocity (V_w) is assumed as representative of average wind conditions for simulation of heat transfer from outer surface of glass envelope to the surroundings. To complete constraint equations, the value of heat loss rate is written in two different ways. Receiver heat loss rate $\dot{Q}_l$, can be written in terms of a heat-loss coefficient (U_l), that is based on the absorber tube surface area:

$$\dot{Q}_l = \pi D_o L U_l (\overline{T}_t - T_a) \tag{32}$$

and it also is expressed from total radiation flux detracted by useful energy:

$$\dot{Q}_l = S A_c - \dot{Q}_u \tag{33}$$

To study the thermal behavior of the collector, the instantaneous thermal efficiency is defined as the rate at which useful energy is delivered to the working fluid divided by the beam solar flux at the collector aperture: $\eta_{th} = \dot{Q}_u / (I_b A_c)$.

Since pressure drop through absorber tube is necessary for calculating the exergy destructed due to viscous fluid, ΔP is modeled using Darcy equation ($\Delta P = 2f(L/D_i)(G^2/\rho)$), resulting pressure drop as a function of mass flow rate, inner tube diameter, tube length and fluid properties, where f and G are friction coefficient and mass flow rate per unit area respectively.

$$\Delta P = \Delta P(\dot{m}_f, D_i, \rho, \mu, f, L, ..) \tag{34}$$

Proper quantification of the heat loss from the receiver is important for simulating the performance and hence designing the collectors.

Optimization procedure and results

Having completed the design analysis, using the optical and thermal analysis in the presence of design goals, constraints including environmental conditions and its corresponding technology to design and manufacture collector, optimization procedure is implemented. Using Lagrange multipliers method, the procedure finds the optimum design point at which exergy efficiency is maximized. Then, based on the material, solar intensity and so on, the designer will determine the performance parameters including $T_{in,f}$, $\dot{m}_f$, and geometrical structure parameters including concentration ratio C and glass envelope diameter D_g, that are incorporated into the design. The objective function, exergy efficiency in Eq. (19), with constraint equations modeled in thermal and optical analysis, Eqs. (20)-(34), form optimization equations. Then, the optimization equations system encompasses 15 equations (Eqs. (20)-(34)) with 19 variables, consequently forms a four-degree of freedom optimization. Optimization variables includes $T_{in,f}$, $\dot{m}_f$, C , D_g, W, L, $\dot{Q}_u$, $T_{out,f}$, F_R, F', U_l, U_c, S, γ, ψ_1, ψ_2, σ^*, β^*, d^* and fundamentally exergy efficiency is a function of these variables. In Lagrange multipliers method [21], the optimum design can be determined from the solution for the following equations:

$$\nabla \eta_E - \sum_{m=1}^{15} \lambda_m \nabla \psi_m = 0 \qquad (35)$$

$$[\psi_k]_{k=1\ldots m} = 0 \qquad (36)$$

where ∇ is the gradient operator and $\psi_1,\ldots,\psi_m$ are constraint equations (20) to (34) respectively.

The method is applied to a typical practical application, we assume a receiver tube of 41.35 mm outer diameter. Selection of such a tube size has no effect on the optimum exergy collected and it just changes the scale of concentrator while concentration ratio is identical by assuming any absorber tube diameter. Selection of a large size tube gives large quantity in aperture width on concentrator and vice versa. Table (1), presents defined parameters (known and unknown parameters) to run the procedure

Table 1: The specifications of a typical solar collector.

absorber tube	
D_o , outer diameter	41.3 mm
D_i , inner diameter	38.1 mm
L , length	Variable
$\overline{T}_t$, mean temperature	Variable
α , absorptivity	0.95
ε_t , emissivity	0.25
reflector concentrator	
A_c , surface area	500 m^2
W , width	Variable
L , length	Variable
C , concentration ratio	Variable
φ_r , rim angle	90 ^{o}
ρ , spectacular reflectivity	0.85
environmental conditions	
T_a , temperature	300 k
P_a , pressure	10^5 Pa
T_s , sun's temperature	5762 k
I_b , solar intensity	700 w/m^2
V_w , wind velocity	5 m/s
glass envelope	
D_g , diameter	variable
P_g , inside pressure	10^5 Pa
ε_g , emissivity	0.88
τ , transmissivity	0.85
optical parameters & errors	
σ_{tot} , total standard deviation	11.3 mrad
β , reflector misalignment	1.0 ^{o}
d_r , miss-location error	6.2 mm
γ , intercept factor	Variable
Initial values of main optimization variables	
$T_{in,f}$, inlet fluid temp.	350 K
$\dot{m}_f$, mass flow rate	1.0 kg/s
C , concentration ratio	10
D_g , glass envelope diameter	60 mm

For a typical collector with surface area of $500\ \mathrm{m}^2$. Once data for design become available, the design method can be made computer-based and be implemented.
The equations system is highly non-linear and is numerically solved by Newton-Raphson method. The main optimization variables are assigned as $T_{in,f}$, $\dot{m}_f$, C and D_g whose initial values to start numerical solution of the procedure were given in table (1). They are important parameters that effects of them in previous works were studied on exergy or thermal efficiency singly, while in the present paper they are studied together. The resulted design parameters are listed in table (2), which are the characteristic of the collector at optimal point. Having concentration ratio, length and width of parabolic reflector, the focal length become $a^{opt} = 0.418\,\mathrm{m}$ and the best profile of concentrator is extracted as $y = 0.597x^2$, and the diameter of glass envelope surrounding the absorber tube becomes 63.38 mm.

Table 2: The results at optimum design point and operating conditions.

η_E^{opt}, exergy efficiency	18.6 percent
η_{th}^{opt}, thermal efficiency	43.05 percent
$T_{in,f}^{opt}$, inlet temperature	481.9 k
$T_{out,f}^{opt}$, outlet temperature	521.78 k
$\dot{m}_f^{opt}$, oil mass flow rate	1.386 kg/s
C^{opt}, concentration ratio	12.58
D_g^{opt}, glass envelope dia.	63.38 mm
a^{opt}, focal length	0.418 m
L^{opt}, length	298.5 m
W^{opt}, width	1.675 m

Fig. (2) shows the variation of exergy efficiency (η_E) with respect to the inlet temperature and concentration ratio i.e. $T_{in,f}, C$ at a three dimensional surface while two other degrees of freedom $(\dot{m}_f, D_g)$ are kept at optimum values. There is only one point representing the optimum design point that η_E reaches maximum value i.e., 18.6 percent.

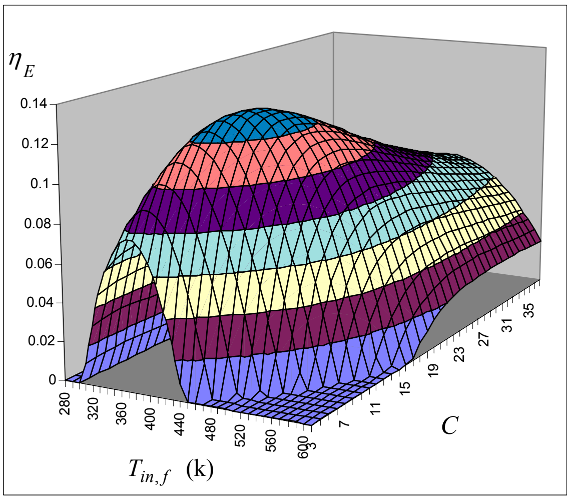

Fig. 2: The variation of exergy efficiency versus inlet temperature and concentration ratio.

Figs. (3-a,b) illustrate the variation of objective function (η_E) when two main variables change coincidently and the others are in optimum values. In fig. (3-a), the value of η_E suddenly rises while C increases and then reaches optimum value. With further increase in C, η_E decreases slightly and optimum points of each curve are located approximately on a line. In fig. (3-a) in which $T_{in,f}$ changes from 300 to 600 exergy efficiency increases with increasing $T_{in,f}$ until it reaches optimum point i.e 481. K. According to the eq. (5), higher degrees in inlet temperature results higher degrees in outlet temperature and consequently in exergy outlet. However, increase in $T_{in,f}$ causes more exergy leakage rate to the surroundings, i.e. $\sum \overset{*}{E}_{loss}$. So, further increase in $T_{in,f}$ beyond optimum point gives lower exergy efficiency as it is obviously appearing in this fig.
Fig. 3-b demonstrate that η_E has a maximum value when mass flow rate $(\dot{m}_f)$ and glass envelope diameter (D_g) change and maximum values of curves are located on a line.

The sensitivity of optimum point to variables change is studied in Fig. 4. It also illustrates the magnitude of each exergy loss and destructed while one of four variables changes. The magnitude of exergy fractions can be compared to each other as shown at optimum point when $\eta_E = 0.186$.

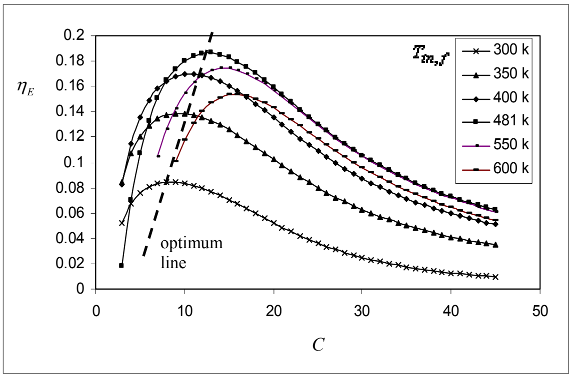

Fig. 3-a: The variation of exergy efficiency against C and $T_{in,f}$.

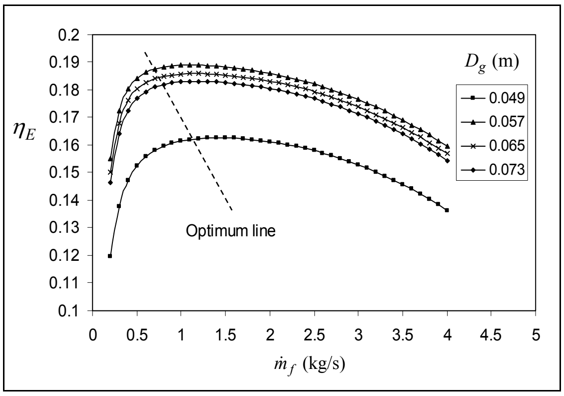

Fig. 3-b: The variation of exergy efficiency against $\dot{m}_f$ and D_g.

The largest values of exergy losses at this point belong to exergy destructed by absorption heat transfer in the first order and exergy loss due to optical errors is in the second order with a very low difference below the first. Heat leakage loss is located in the third order and exergy destructed by pressure drop and conduction heat transfer are approximately negligible at optimum design point.
Fig. 4-a shows the sensitivity of exergy efficiency and fractions to $T_{in,f}$. Whereas the exergy collected monotonically increases with an increase in collector inlet temperature, the exergy collected increase to a maximum value, beyond which further temperature increase cause a decrease in exergy collection.

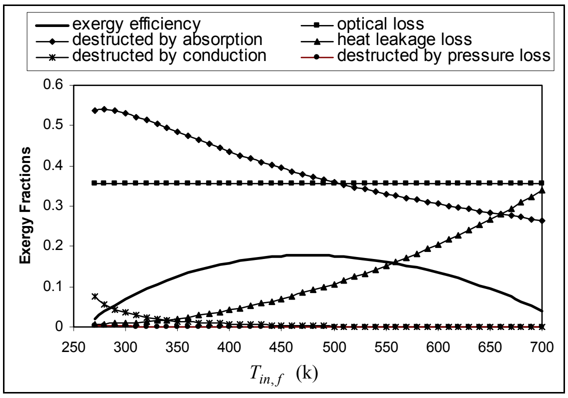

Fig. 4-a: The variation of exergy fractions versus collector inlet temperature.

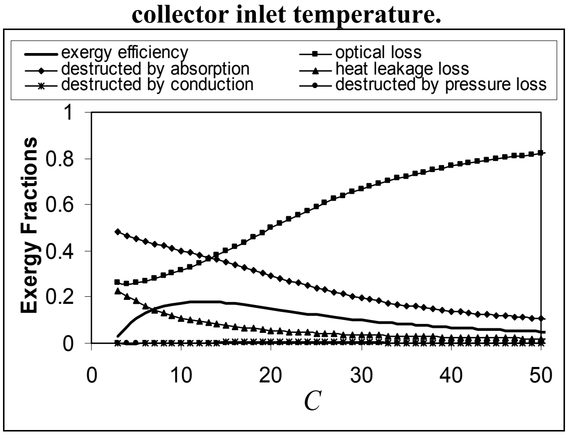

Fig. 4-b: The variation of exergy fractions versus collector concentration ratio.

Once a collector has been optimized for operation at a certain temperature, the deviation from optimal performance at a different temperature is negligible. In a range of $\pm 50\ ^{\circ}\mathrm{C}$ in inlet temperature around the optimum point, there is a very small change in exergy efficiency. The same change happens when mass flow rate $\dot{m}_f$ changes in Fig. 4-c. The results give optimum value of 63.33 mm for glass envelope diameter i.e. the gap width becomes 22 mm. Results in Fig. 4-d demonstrate that low values of glass diameter cause significant losses in exergy efficiency while increasing in D_g beyond optimum point gives a very slight fall in exergy efficiency.

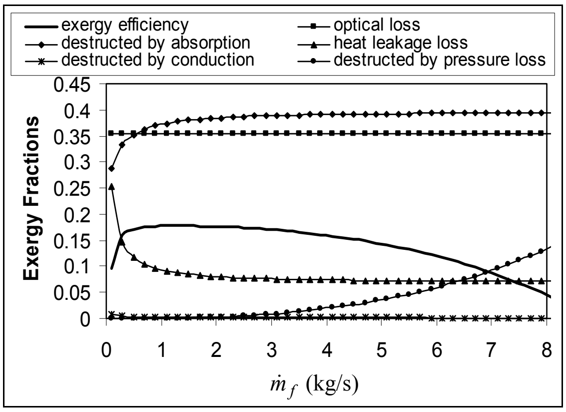

Fig. 4-c: The variation of exergy fractions versus mass flow rate.

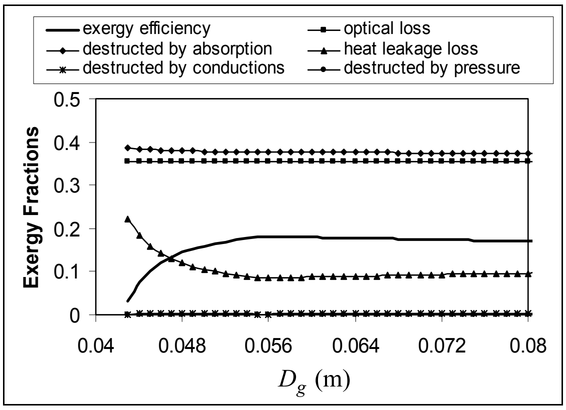

Fig. 4-d: The variation of exergy fractions versus glass envelope diameter.

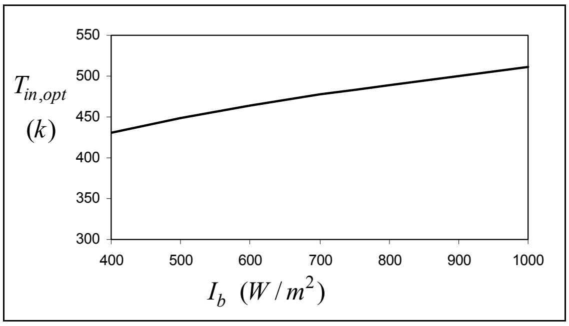

Fig. 5-a: The variation of optimum values of inlet temperature versus I_b .

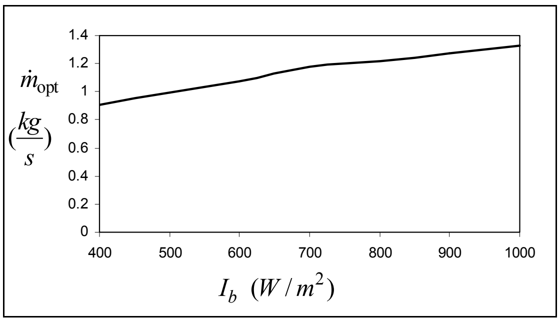

Fig. 5-b: The variation of optimum values of mass flow rate versus I_b .

The deviation from optimum point due to changes in I_b was investigated by authors in a range of 400 to 1000W/m^2. Results are shown in Fig 5 and demonstrate that an increase in I_b gives higher values in calculated optimum point for both $T_{in,f}$ and $\dot m_f$. $T_{in,f}^{opt}$ Changes from 430 to 508 K and $\dot m_f^{opt}$ changes from 0.91 to 1.327kg/s. C^{opt} also slightly decreases from 13.2 to 1.1 and optimum values of $D_g{}^{opt}$ is approximately constant, and exergy efficiency decreases about 6 percent in this range.

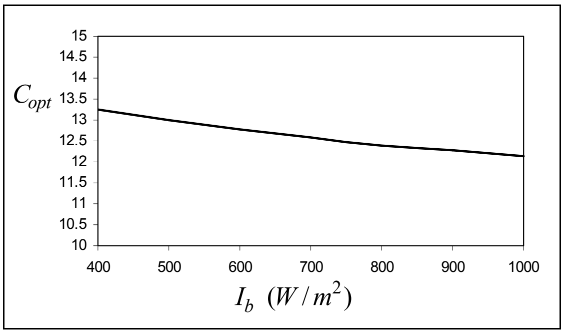

Fig. 5-c: The variation of optimum values of concentration ratio versus I_b .

Although these changes in the calculated optimum values of both $T_{in,f}$ and $\dot m_f$ are not considered significant in such a wide range of I_b, both of these performance parameters can be set at corresponding optimum values by devising a control system. To reach the best design, optimum values must be calculated at the dominant value of I_b in each environment.

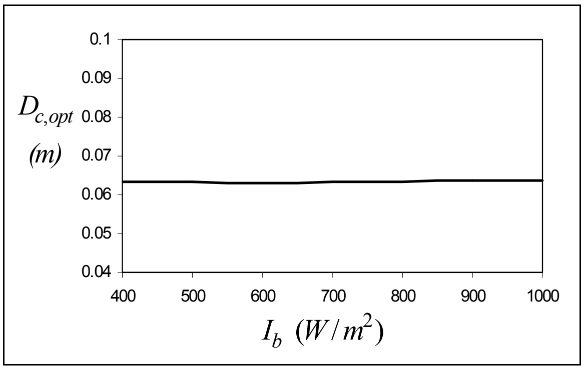

Fig. 5-d: The variation of optimum values of glass envelop diameter versus I_b .

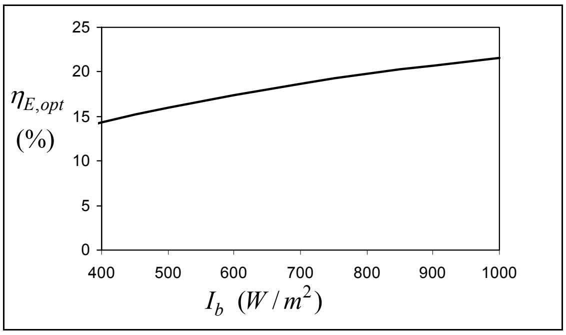

Fig. 5-e: The variation of optimum values of exergy efficiency versus I_b.

Conclusion

In this paper, a new and comprehensive method for defining design and operating conditions for linear parabolic solar collectors was presented. The proposed method consists of objective design approach, and simulation of thermal and optical performance of collectors. Exergy efficiency was introduced as objective function and regarding thermal and optical aspects, mathematical modeling for real performance of the collector implemented. In optimization procedure, optimum values of collector inlet temperature, mass flow rate, concentration ratio and glass envelope diameter as main variables, were extracted in the presence of environment conditions, material selected and design constraints. Applying the method to a typical collector, the optimization procedure using Lagrange multipliers method found the optimum design and operating conditions so that the maximum value of exergy efficiency of the collector was gained. Results prove that for each of four optimization degree of freedom, there is just one point at which collected exergy reaches peak value. The variation of exergy efficiency and the share of exergy losses around optimum point and the sensitivity of optimization variables $(T_{in,f}, \dot{m}_f, C, D_g)$ on the exergy efficiency and losses were evaluated.

Nomenclature

A_c : collector surface area (m^2)
a : focal length
C : Concentration ratio
c_p : oil heat capacitance ($\mathrm{kJ/kg.K}$)
D_i : absorber tube inner diameter (m)
D_o : absorber tube outer diameter (m)
D_g : diameter of the concentric glass envelop (m)
d_r : concentrator miss-location error parameter (mm)
$\dot{E}$: exergy rate (W)
F_R : Heat removal factor
F' : collector efficiency factor
I_b : beam solar intensity ($\mathrm{W/m^2}$)
L : Concentrator length (m)
$\dot{m}_f$: mass flow rate (kg/s)
P : pressure (Pa)
$\dot{Q}$: Heat transfer rate (W)
S : absorbed flux (w/m^2)
T : temperature (K)
$\overline{T}_t$: absorber mean temperature
U_c : overall heat gain coefficient ($\mathrm{W/m^2K}$)
U_l : overall heat loss coefficient ($\mathrm{W/m^2K}$)
W : concentrator aperture width (m)
α : absorber tube absorptivity for solar radiation
β : reflector misalignment and tracking error angle (degree)
ε_g : glass envelope emissivity
ε_t : absorber tube emissivity
η_o : optical efficiency
η_E : exergy efficiency
η_P : Petela efficiency
ρ : spectacular reflectivity of the concentrator surface
σ_{tot} : total standard deviation of the reflected energy distribution (mrad)
φ_r : rim angle (degree)
τ : glass envelope transmissivity for solar radiation
γ : intercept factor

Subscripts

in : Inlet
out : Outlet
a : Ambient

s : Solar
f : fluid flow
des : Destruction
l : Loss
u : useful energy

Superscript

opt : values of parameters at optimum design point
*: non-dimension parameters

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بهینه سازی اگزرژی متمرکز کننده های خورشیدی از نوع سهموی خطی

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چکیده

در این مقاله، یک روش جدید بهینه سازی متمرکزکنندههای خورشیدی از نوع سهموی خطی ارائه می شود. ابتدا یک مدل سازی جامع ریاضی و شبیه سازی حرارتی و اپتیکی از عملکرد متمرکزکننده انجام و پارامترهای هندسی و ترمودینامیکی به عنوان متغیرهای بهینه سازی در نظر گرفته می شوند. با به کار بردن رابطه بدست آورده شده برای راندمان اگزرژی کلکتور، میزان افت های اگزرژی و شرایط طراحی و کارکردی بهینه مورد تحقیق قرار می گیرند. تابع موضوع بهینه سازی که همان راندمان اگزرژی می باشد به همراه معادلات محدودیت ها، یک سیستم بهینه سازی با درجه آزادی چهار را تشکیل می دهند. با استفاده از روش ضرایب لاگرانژ، رویه بهینه سازی بر روی یک کلکتور خورشیدی نمونه اجرا و نقطه طراحی بهینه استخراج می شود. مقادیر بهینه شامل "دمای ورودی کلکتور"، "نرخ جریان جرمی روغن"، "نسبت تمرکز" و "قطر پوشش شیشه ای" به طور همزمان با حل سیستم معادلات غیر خطی مدل شده محاسبه می شوند. برای مطالعه تاثیر تغییرات متغیرهای بهینه سازی بر روی اگزرژی جمع آوری شده کلکتور، حساسیت نقطه بهینه به تغییر پارامترهای کلکتور و شرایط عملکردی برآورد می گردد و تغییرات اگزرژی های جزء در نقطه بهینه بررسی می گردند.

واژه های کلیدی : اگزرژی - بهینه سازی - کلککتور سهموی خطی - انرژی خورشیدی