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$$\dot{\epsilon}_{xx} = \frac{\partial U}{\partial x} = \frac{\partial^2 \phi}{\partial x \partial y}$$

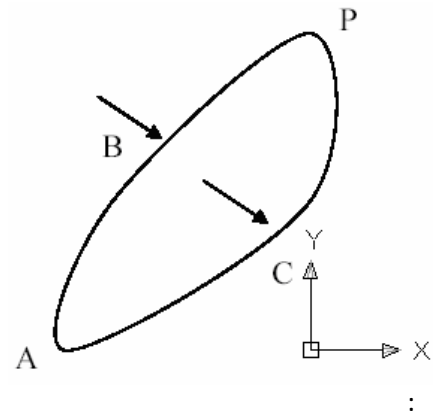
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$$\dot{\epsilon}_{yy} = \frac{\partial V}{\partial y} = -\frac{\partial^2 \phi}{\partial x \partial y}$$

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$$\dot{\epsilon}_{xy} = \frac{1}{2} \left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right)$$

()



$\dot{\epsilon}_{yy}$ x

$\dot{\epsilon}_{xx}$
 $\dot{\epsilon}_{xy}$ y

$$U = \frac{\partial \phi}{\partial y}$$

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$$\dot{\epsilon}_{xx} = -\dot{\epsilon}_{yy}$$

$$V = -\frac{\partial \phi}{\partial x}$$

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τ_{yz} τ_{xz}

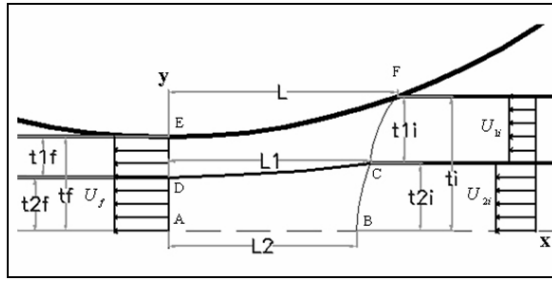
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$$\sigma_z = \frac{\sigma_x + \sigma_y}{2}$$

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$$\dot{\epsilon}_{eq} = \sqrt{\frac{2}{3} (\dot{\epsilon}_{xx}^2 + \dot{\epsilon}_{yy}^2 + 2\dot{\epsilon}_{xy}^2)}$$

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$$(t_0 b_0 U_0) = Q'$$

$$(t_0 b_0 U_0 = t b U = Q')$$

$$U = \frac{Q'}{t b} = \frac{Q}{t}$$

b t

b₀ t₀

$$\phi = U_m t$$

(CDEF)

$$\phi_1 = U_{m1} [(y - y_2) + A_1(x)(y - y_1)(y - y_2)]$$

$$U_{m1} = \frac{\phi_{y1} - \phi_{y2}}{y_1 - y_2} = \frac{Q_1}{y_1 - y_2}$$

$$Q_1 (y_1 - y_2) = t_1$$

$$\dot{W}_{id} = \int_v \sigma \dot{\epsilon}_{eq} dv$$

$$\dot{W}_{fr} = \int_{s_f} \tau_{xy} |\Delta u|_{s_f} ds_f$$

$$\dot{W}_{sh} = \int_{s_s} \tau_{xy} |\Delta u|_{s_s} ds_s$$

$$J = \dot{W}_{id} + \dot{W}_{fr} + \dot{W}_{sh}$$

$$J = \dot{W}_{id} + \dot{W}_{fr} + \dot{W}_{sh}$$

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$$J = \dot{W}_{id} + \dot{W}_{fr} + \dot{W}_{sh}$$

$$y_2' = y_2'(x) = 2 \frac{t_{2i} - t_{2f}}{L_1^2} x \quad () \quad y_1 \quad (EF)$$

$$t_1 = y_1 - y_2 \quad () \quad y_2 \quad (CD)$$

$$y_1 = y_1(x) = R + t_f - \sqrt{R^2 - x^2} \quad ()$$

$$y_2 = y_2(x) = \frac{t_{2i} - t_{2f}}{L_1^2} x^2 + t_{2f} \quad ()$$

$$\phi_3 = U_{li}(y - t_{2i}) \quad () \quad \phi_{y2} \quad \phi_{y1}$$

$$A_1(x) = a_1 x^2 + b_1 \quad ()$$

$$\phi_1 = \phi_3 \quad (CF) \quad R \quad t_f \quad b_1 = 0$$

$$y_3 = y_3(x) = \frac{-B_{li} + \sqrt{B_{li}^2 - 4A_{li}C_{li}}}{2A_{li}} \quad J \quad a_1$$

$$A_{li} = A_1(x) \quad \phi_{11} = U_{m1}(y - y_2)$$

$$B_{li} = 1 - A_1(x)(y_1 + y_2) - \frac{y_1 - y_2}{t_{li}}$$

$$C_{li} = A_1(x)y_1y_2 - y_2 + \frac{t_{2i}}{t_{li}}(y_1 - y_2) \quad \phi_{12} = U_{m1}[A_1(x)(y - y_1)(y - y_2)]$$

$$U_1 = \frac{\partial \phi_1}{\partial y} = U_{m1}[1 + A_1(x)(2y - (y_1 + y_2))] \quad ()$$

$$U_1 \sin \theta - V_1 \cos \theta = U_{li} \sin \theta \quad (y_3) \quad V_1 = -\frac{\partial \phi_1}{\partial x} = U_{m1} \frac{(y_1' - y_2')}{(y_1 - y_2)} [(y - y_2) + A_1(y - y_1)(y - y_2)] - U_{m1}[-y_2' + A_1'(y - y_1)(y - y_2) - A_1y_1'(y - y_2) - A_1y_2'(y - y_1)] \quad ()$$

$$U_{li}t_{li} = U_1t_1 = U_{1f}t_{1f} = Q_1 \quad () \quad A_1'(x) = 2a_1x \quad ()$$

$$U_{1f}, U_{li} \quad \theta = \tan^{-1} y_3'(x) \quad () \quad y_1' = \frac{x}{\sqrt{R^2 - x^2}} \quad ()$$

$$U_2 = \frac{\partial \phi_2}{\partial y} = U_{m2}[1 + A_2(x)(2y - y_2)] \quad (\text{DE})$$

$$V_2 = -\frac{\partial \phi_2}{\partial x} = U_{m2} \frac{y_2'}{y_2} [y + A_2(x)y(y - y_2)] - U_{m2}[A_2'(x)y(y - y_2) - A_2(x)y_2'(y)] \quad () () ()$$

$$A_2'(x) = 2a_2x \quad () \quad \dot{\epsilon}_{xx1} + \dot{\epsilon}_{yy1} = 0 \quad ()$$

$$t_2 = y_2 \quad () \quad \dot{\epsilon}_{xx1} = -\dot{\epsilon}_{yy1} \quad ()$$

$$\phi_4 = U_{2i}y \quad ()$$

$$y = 0 \quad y_2 \quad y_2 \quad y_1 \quad (\text{ABCD})$$

$$\phi_2 = \phi_4 \quad () \quad \phi_2 = U_{m2}[y + A_2(x)y(y - y_2)] \quad ()$$

$$y_4 = y_4(x) = y_2 + \frac{t_{2i}}{A_2(x)} \quad () \quad U_{m2} = \frac{\phi_{y2} - \phi_{y0}}{y_2} = \frac{Q_2}{y_2} \quad ()$$

$$U_{2i}t_{2i} = U_2t_2 = U_{2f}t_{2f} = Q_2 \quad () \quad y_2 = t_2 \quad Q_2$$

$$A_2(x) = a_2x^2 + b_2 \quad () \quad U_{2f}, U_{2i} \quad ()$$

$$(\text{AD}) \quad b_2 = 0 \quad a_2$$

$$() () () \quad J$$

$$\dot{\epsilon}_{xx2} + \dot{\epsilon}_{yy2} = 0 \quad ()$$

()

$$\dot{\epsilon}_{xx2} = -\dot{\epsilon}_{yy2}$$

J

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($R, \omega, \sigma_1, \sigma_2, t_{i1}, t_{i2}, t_f, m_1, m_2$)

($a_1, a_2, L_1, Q_1, t_{2f}$)

[]

(Q_1)

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(Q_2)

$$F = \frac{J - [(T_{b1} \cdot t_{i1} + T_{b2} \cdot t_{i2}) \cdot R \cdot \omega]}{L \cdot \omega}$$

$$U_{1f} = U_{2f} = U_f \Rightarrow \frac{Q_1}{t_{1f}} = \frac{Q_2}{t_{2f}}$$

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(Back

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F Tension Force)

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$$F = \frac{J}{L \cdot \omega}$$

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$$L = \sqrt{2 \cdot R \cdot t_i \cdot r}$$

L

$$\tau_{xy1} = m_1 k_1 \quad \tau_{xy2} = m_2 k_1$$

()

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r

$m_2 \quad m_1$

$$r = 1 - \frac{t_f}{t_i}$$

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$$k_1 = \frac{\sigma_1}{\sqrt{3}}$$

()

$$r_1 = \frac{(t_{i1} - t_{1f})}{t_{i1}}$$

()

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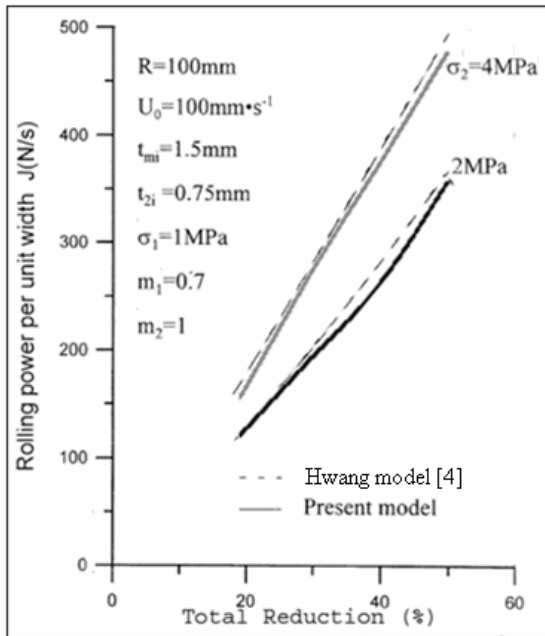
(y_4, y_3)

(%)

$$r_2 = \frac{(t_{2i} - t_{2f})}{t_{2i}}$$

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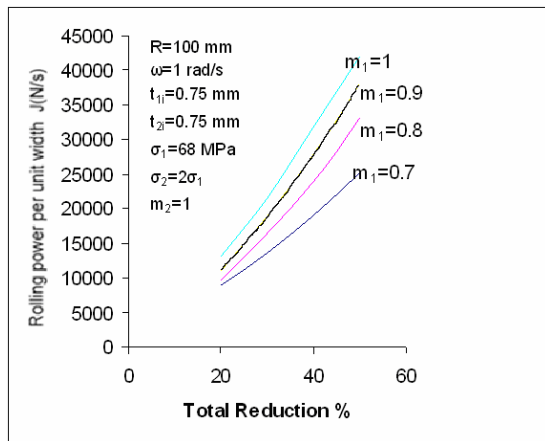
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($m_2=1$)

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$$P_m = \frac{F}{L_1}$$

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P_m
 L_1

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(l)

(y_2)

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$$U_{1@y_2} = U_{m1}[1 + A_1(x)(y_2 - y_1)]$$

$$U_{2@y_2} = U_{m2}[1 + A_2(x)y_2]$$

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$$U_{m1}[1 + A_1(y_2 - y_1)] = U_{m2}[1 + A_2 * y_2]$$

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(l)

x

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(m₁)

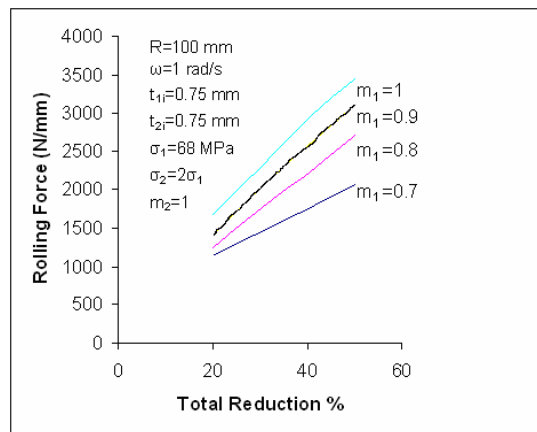
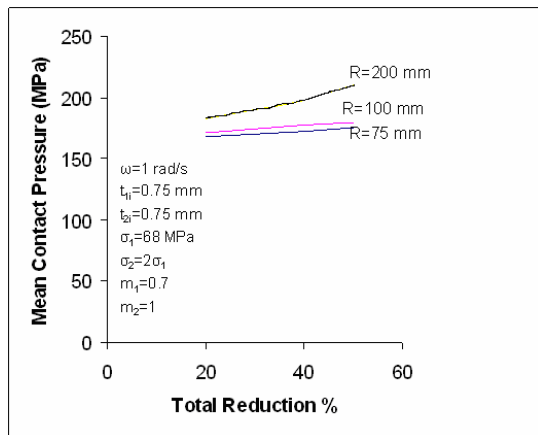
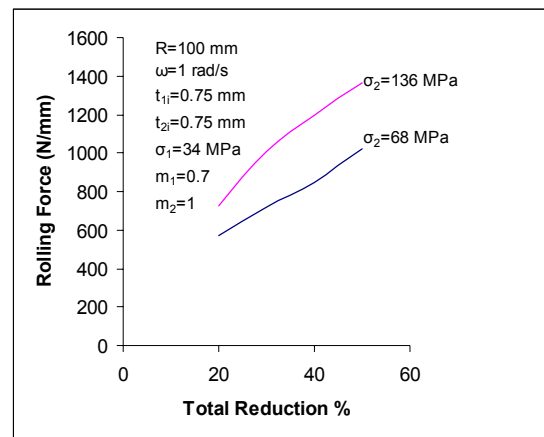
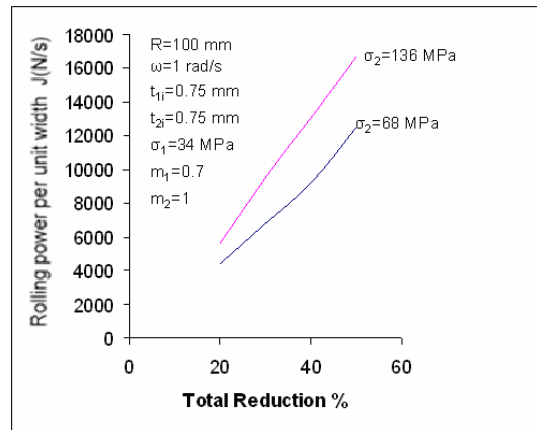
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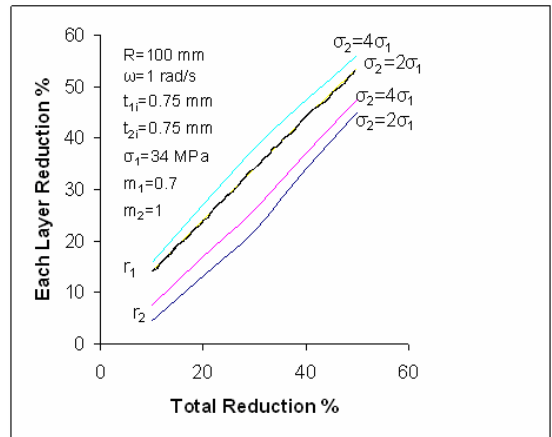
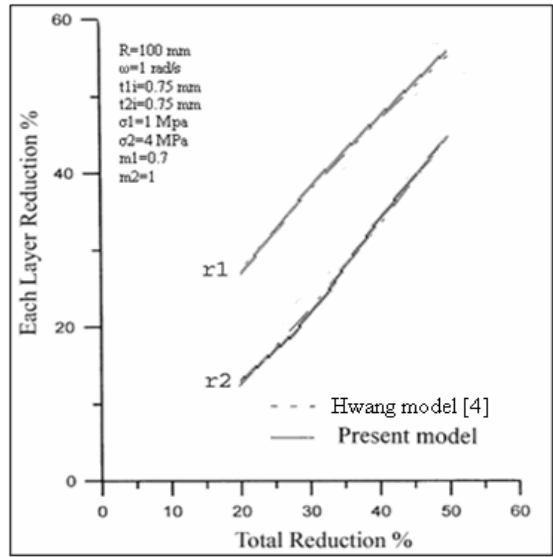
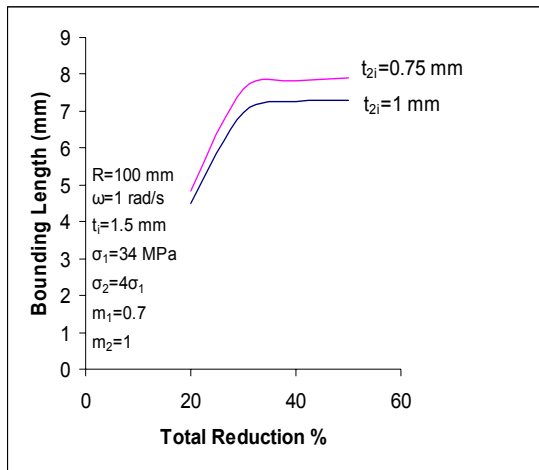
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		$: m_2, m_1$	•
		$: P_m$	
		$: Q$	
		$: r$	
		$: R$	
($: s$	
		$: t$	
	x	$: U$	
		$: U_m$	•
		$: v$	
	y	$: V$	
		$: \dot{W}_{id}$	•
		$: \dot{W}_{fr}$	
		$: \dot{W}_{sh}$	
		$: y_2, y_1$	
		$: \Delta u$	
		$: \Phi$	
	x	$: \dot{\epsilon}_{xx}$: b
	y	$: \dot{\epsilon}_{yy}$: F
	xy	$: \dot{\epsilon}_{xy}$: J
		$: \dot{\epsilon}_{eq}$: l
yz xz		$: \tau_{yz} \tau_{xz}$: L
			: L_1
		:	
			: L_2
		:	
	()	: i	
	()	: f	: L_n

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1 - Homogeneous

2 - Isotropic

3 - Rigid-Perfectly Plastic

4 - Steady State

5 - Mean Contact Pressure

6 - Mean Pressure

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